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# Page 25 Exercise 2.1

Suppose we are analyzing a collection of games. Each game falls into one of four categories based on its Primary Genre:

Define the following subsets:

* A: The subset of games with a User Rating Count of 10,000 or more.
* B: The subset of games in the Strategy genre.
* C: The subset of games with an Average User Rating of 4.0 or higher.

Step 1 Compute A, B, C

1. Subset A

Numbers of elements in A: |A|.

Subset B

Number of Elements In B: |B|.

Subset C”

Number of elements in C: |C|

Step 2 Perform Operations

1. Union

Cardinality:

1. Intersection

1. Union ():

| = |A| + |C| - |A|

1. Intersection ():

:

Cardinality:

| = Number of games satisfying both conditions.

1. Union (
2. Intersection ():

Complements of B:

}

Cardinality:

|

Step 3: Substitute Cardinalities

1 Size of A

|A| = 353

Size of B:

|B| = 0

Size of C:

|C| = 5573

Answer

353 + 0 – 0 = 353

1. | = |A| + |C| - |A|

353 + 5573 – 327 = 5599

1. () = 0
2. () = 5573 − 0 = 5673

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# Exercise 2.10

The dataset contains information about mobile games, and we want to analyze their probabilities based on certain criteria. Suppose the sample space consists of five simple events corresponding to the Primary Genre of the games:

Where:

* E1​: Games in the **Strategy** genre
* E2​: Games in the **Puzzle** genre
* E3​: Games in the **Board** genre.
* E4​: Games in the **Simulation** genre.
* E5​: All other genres combined.

Part (A):

If:

P () = P ( = 0.15, P () = 0.4, P () = 2P ()

Find the probabilities of and .

Part (B):

If:

And the remaining probabilities ( )) are equally probable, find the probabilities of .

Part (A) Solution

We are given P () = P ( = 0.15, P () = 0.4, P () = 2P ()

Total probability for the sample space must equal 1:

Substitute the known probabilities

0.15 + 0.15 + 0.4 + P() + P() = 1

We also know that:

P () = 2P(()

Let P(() = x. Then P () = 2x

Substitute these into the equation:

0.15 + 0.15 + 0.4 + 2x + x = 1

Simplify:

0.7 + 3x = 1

Solve for x:

3x = 0.3 🡺 x = 0.1

Thus

P(() = 0.1, and P () = 2x = 0.2

Part B:

We are given:

And the remaining probabilities ()) are equally probable

. Then;

3y = 0.3

Solve for y

🡺

.

The remaining probabilities ()) must be equal, and their sum must satisfy:

Substitute the known values:

1− 0.3 − 0.1 = 0.6

Let Then:

3z = 0.6 🡺 z = 0.2

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# Exercise 2.28

A company is randomly selecting two games from a list of four equally qualified games. One game is from the **Strategy genre**, and the remaining three games ) are from the **Puzzle genre**. The company randomly selects two games for a special promotional feature.

1. **(a)** List all the possible outcomes of this experiment.
2. (b) Assign reasonable probabilities to the sample points.
3. **(c)** Find the probability that the game from the **Strategy genre** is selected.

Part (a): List the possible outcomes

We are choosing 2 games out of 4. Using the combination formula:

The sample space S contains all combinations of 2 games:

, ) ,

Part (b): Assign probabilities

The games are equally qualified, and the selection is random. Therefore, all outcomes are equally likely. The probability of each outcome is:

Each sample point has a probability of .

Part (c): Probability of selecting the Strategy game

We are interested in the event where the game from the Strategy genre ) is selected. The favorable outcomes are: , ) ,}

The number of favorable outcomes is 3. The probability of being selected is:

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# Exercise 2.2

A company is running a promotional contest based on three randomly chosen game genres from the dataset:

1. One game genre represents the **Strategy** genre (), which is the "Good Prize."
2. The other two game genres, **Puzzle** (), and Simulation ) represent "Dud" prizes.

The contest proceeds as follows:

1. **(a)** If a participant selects a genre at random, calculate the probability that they select the **Strategy** genre.
2. **(b)** After the participant selects one genre, the host reveals one of the dud genres and offers the participant the chance to switch their choice. Should the participant switch? Calculate the probabilities to justify your answer.

Part (a): Probability of selecting the Strategy genre

The sample space consists of the three genres:

, }

Where:

G: Strategy

: Puzzle

: Simulation

Since the participant selects a genre at random, each genre has an equal probability:

Thus, the probability of selecting the Strategy genre is:

Part (b): Should the participant switch?

To analyze whether the participant should switch, we consider the following:

1. Initial probabilities:

The probabilities of initially selecting each genre are:

1. Host reveals a dud:

After the participant selects one genre, the host reveals one of the "Dud" genres. The host knows which genre contains the good prize (G) G) and will always reveal a dud.

Case 1: Participant initially chooses G (Good Prize):

If the participant chose G the host can reveal either or . The probability of winning by sticking with G remains:

Case 2: Participant initially choose or :

If the participant chose , the host will reveal (or vice versa). Switching in this case guarantees that the participant will win the good prize (G) The combined probability of winning by switching is:

1. Conclusion:

The probability of winning by sticking with the initial choice is , while the probability of winning by switching is

The participant should always switch to maximize their chances of winning.

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# Exercise 2.146

A company is running a promotional campaign in which 5 games are randomly selected from a list of 52 games in the dataset. Each game belongs to one of 4 genres: Strategy, Puzzle, Simulation, or Board. What is the probability that all 5 selected games are from the same genre?

Step 1: Total Ways to Choose 5 Games

From a list of 52 games, the total number of ways to choose 5 games is given by the combination formula:

Thus, there are 2,598,9602,598,9602,598,960 ways to choose any 5 cards.

Step 2: Ways to Choose 5 Cards of the Same Suit

Each suit (e.g., Spades, Hearts, Clubs, Diamonds) contains 13 cards. To choose 5 cards from a single suit, the number of combinations is:

Since there are 4 suits, the total number of ways to choose 5 cards of the same suit is:

Step 3: Probability of All 5 Cards Being of the Same Suit

The probability is the ratio of favorable outcomes (5 cards of the same suit) to the total outcomes (any 5 cards):

The probability that all 5 cards are of the same suit is approximately:

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# Exercise 2.35

A company offers a promotional campaign in which customers can choose one game genre followed by one specific game from that genre. The dataset contains 4 genres (Strategy, Puzzle, Simulation, Board), with each genre containing 13 games (like a deck of cards).

If a customer selects one game genre and then one game from the chosen genre, how many different promotional combinations can the company offer?

Step 1: Total Choices

1. Number of genres available:
   1. There are 444 genres: **Strategy**, **Puzzle**, **Simulation**, and **Board**.
2. Number of games in each genre:
   1. Each genre contains 13 games

Step 2: Total Promotional Combinations

For each genre, a customer can choose from 13 games. Therefore, the total number of combinations is:

Substitute the values:

Final Answer:

The company can offer 52 different promotional combinations, where a customer selects one genre followed by one specific game.

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# Exercise 2.101

A company uses a two-stage review process to approve games for a promotional campaign. The first reviewer approves a game with a probability of 0.1. If a game passes the first review, the second reviewer approves it with a probability of 0.5. What is the probability that a game gets approved by both reviewers?

To calculate the probability of a game being approved by both reviewers, we use the multiplication rule of probability, which states:

Step 1: Assign Probabilities

The probability that the first reviewer approves the game is:

The probability that the second reviewer approves the game, given that the first reviewer approved it, is:

Step 2: Multiply Probabilities

Using the multiplication rule:

Substitute the values:

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## Exercise 3.78

In a dataset of games, 60% of the games belong to the **Strategy** genre (G). If games are randomly selected and inspected, what is the probability that:

* 1. Exactly five games are inspected before encountering the first game from the Strategy genre?
  2. At least five games are inspected before encountering the first game from the Strategy genre?

This is a geometric probability problem, where the probability of encountering the first success (a game from the Strategy genre) on the kth trial is given by:

Where:

p = 0.6 (probability of selecting a Strategy game)

1 – p = 0.4 (probability of selecting a game that is not Strategy)

Part (a): Probability that exactly five games are inspected before encountering the first Strategy game

Substitute k=5 into the formula:

Simplify:

Part (b): Probability that at least five games are inspected before encountering the first Strategy game

The probability of at least k trials before the first success is:

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A promotional campaign randomly selects a game from a dataset containing 52 games. During the promotion, 15 of the games are unavailable due to prior selection or technical issues. What is the probability that a randomly selected game is available?

Step 1: Total Games and Unavailable Games

Total games in the dataset: 52

Unavailable games: 15

Available games: 52 – 15 = 37

Step 2: Calculate the Probability of Selecting an Available Game

The probability of selecting an available game is the ratio of available games to the total number of games:

Substitute the values:

Final Answer:

The probability that a randomly selected game is available is

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## Exercise 5.9

Let and represent two attributes of games in the dataset:

: The rating of a game (between 0 and 1).

: The price of a game (between $0 and $1).

The joint probability density function for and is given as:

1. Find the value of k that makes a valid probability density function.
2. Find the probability that a randomly selected game has and

Part (a): Finding k:

The total probability for a valid joint probability density function must equal 1:

Substitute

Evaluate each term:

Combine:

Substitute back:

🡺 k = 2

Part (b): Finding )

The probability is the integral of over the specified bounds:

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In a dataset of games, 40% of the games belong to the Strategy genre and 60% belong to the Puzzle genre. It is reported that 30% of the Strategy games and 70% of the Puzzle games have a user rating of 4.5 or higher. If a randomly selected game is found to have a rating of 4.5 or higher, what is the probability that it belongs to the Puzzle genre?

To solve, we use Bayes' Theorem:

Step 1: Calculate

The total probability of a game having a rating of 4.5 or higher is:

P(Rating≥4.5∣Strategy)=0.3

P(Rating≥4.5∣Puzzle)=0.7

P(Strategy)=0.4

P(Puzzle)=0.6

P (Rating ≥ 4.5) = ( 0.3⋅0.4) + (0.7⋅0.6)

Calculate:

Step 2: Calculate

Using Bayes' Theorem:

=0.778

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## Exercise 3.7

In a dataset of games, there are 3 genres (**Strategy**, **Puzzle**, **Simulation**), and 3 new games are randomly assigned to these genres. Let Y represent the number of genres that remain empty after assigning the 3 games. Find the probability distribution for Y, the number of empty genres.

Step 1: Total Possible Outcomes

Each of the 3 games can be assigned to any of the 3 genres, so the total number of possible outcomes is:

Step 2: Possible Values of Y (Number of Empty Genres)

1. Y=0 (No empty genres): This occurs when each genre has at least one game. The only way this can happen is by distributing the 3 games across all 3 genres without leaving any empty.

Using the principle of inclusion-exclusion, the number of arrangements is:

Probability:

1. **Y=1 (1 empty genre):** This occurs when exactly 2 genres have at least one game, and 1 genre is empty. There are 3 ways to choose which genre is empty, and the remaining 3 games can be distributed among the 2 non-empty genres in ways (subtracting cases where all games go into one genre). Therefore:

Probability:

1. **Y=2 (2 empty genres):** This occurs when all 3 games are assigned to the same genre. There are 3 genres, and all 3 games can go into any one of them:

Probability: